Spatial Separation of Multi-User MIMO Channels

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Abstract— **Since multi-antenna (MIMO) systems are becoming more popular thanks to their inherent potential for capacity improvement, interference from MIMO transceivers is an increasingly serious concern. Spatial multiplexing schemes are particularly vulnerable to multi-user interference. Fortunately, this interference can be mitigated, when the channel matrices show a sufficient spatial separation.**

In this paper, we quantify the separability of multi-user MIMO channels using actual measurements in a scenario where a single outdoor base station transmits to two indoor mobile receivers. To quantify the spatial distance between the two users, we compare the spatial correlation matrices using two simple measures: (i) matrix collinearity, and (ii) the condition number ratio. Both measures are directly linked to MIMO system performance.

Our measurement-based evaluations demonstrate that the downlink channels of different users can have a significantly different spatial structure, even when the users are in the same room. This leads to the following conclusions: (i) new multi-user MIMO models are needed to describe the spatial characteristics of different users, and (ii) spatial interference can be well managed by appropriate scheduling and precoding algorithms.

Keywords—**Multi-user MIMO systems; radio channel measurements; multi-user channel modeling;**

I. INTRODUCTION

After close to a decade of research on MIMO and its capacity and throughput benefits, MIMO has finally found its way into many wireless communications standards and commercial products [1], [2]. Due to the capacity enhancements from MIMO schemes, the number of multi-antenna terminals is constantly increasing. Thus, wireless networks employing MIMO technology result in increased levels of interference between terminals. Interference is particularly relevant in MIMO systems [3], and significantly harms spatial multiplexing schemes. On the other hand, MIMO offers a new possibility to distinguish between signal and interference in the spatial domain [4]. By employing interference-aware communication schemes it is possible to mitigate performance impairments, whenever the underlying channel permits.

A first approach of including the spatial properties of the multi-user MIMO channels into a channel estimation algorithm was performed in [5]. The authors used the Kronecker model [6], where they assumed that each mobile station has a spatial receive correlation matrix, while the base station is characterized only by a single transmit correlation matrix. This approach is questionable, since different mobile positions might not lead to the same transmit correlation matrix (i.e. the classical Kronecker assumption might not be valid regarding multi-user channels).

Different correlation matrices at the transmitter would even be quite desirable, since transmission schemes can make use of this separation by appropriate prefiltering methods. So, understanding the propagation channel is a necessary condition to develop effective interference mitigation techniques. To investigate whether the channels from different users are spatially separable, MIMO measurements are inevitable.

To date only a very limited number of *multi-user* MIMO channel measurements have been undertaken to investigate the interference issues with multi-user MIMO channels.[1] These measurements were carried out with a huge effort using two different kinds of channel sounders, which made the evaluation of the data even more cumbersome. However, using two channel sounders yields the highest measurement accuracy of the characteristics of a two-user (i.e. multi-user) MIMO channel. Since already the availability of a single channel sounder is an expensive proposition (let alone two of them), we recently discussed that a single channel sounder is sufficient to emulate multi-user MIMO measurements [8], which sets the premise for the evaluations we do in this paper.

Contribution We compare the spatial separability of multi-user MIMO channels using two similarity metrics: matrix collinearity, and the condition number ratio. These metrics are applied to outdoor-to-indoor MIMO channel measurements in a 2×4 configuration, where the (indoor) receiver was moved along well-defined routes. Based on the results from these metrics we discuss the validity of current multi-user modeling assumptions.

Organization This paper is organized as follows. Sec-

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[1] All of these measurements were part of the Nordic WILATI project [7]

Fig. 1. Outdoor measurement map (Picture: © Google Maps)

tion II presents the multi-user measurements, the equipment, and the environments. In Section III, we describe the similarity measures that we use to quantify the distance between the measured channels. Section IV discusses our results, showing that multi-user channels can indeed show significantly different spatial structure. Finally, Section V presents our conclusions.

II. MEASUREMENTS

In this paper we evaluated the measurement data from the Stanford July 2008 Measurement Campaign[2] [9].

A. Environment

We conducted the measurements in an outdoor-to-indoor base station (BS) to mobile station (MS) environment. Two BS positions were investigated, and at each, the BS was rotated into three different directions (see Figure 1). The indoor environment is a cubicle-style office environment in Santa Clara, California. Indoors, we measured along 5 routes going along the cubicles (see Figure 2). These five routes were measured for every base station position and orientation. A few routes were even measured multiple times with the same BS configuration, by which we could validate the approach of using a single sounder for doing multi-user measurements [8].

To ensure comparable measurements the following precautions were taken: (i) The measurement routes were marked on the floor by duct tape. The sounder was pushed along the taped route as accurately as possible. (ii) For each specific route, all people indoors helping in the measurements were staying at the same position in the room for all different BS configurations. This is to ensure the radio environment to be as similar as possible.

For the numerical evaluations in this paper we chose a set of particularly interesting routes, which are identified as TxpDd-Rr, where p is the transmitter position, d the transmitter direction, and r the route number, e.g. Tx1D2-R3 denotes the measurement of Route 3 (see Figure 2) with the transmitter at position 1 facing into direction 2 (see Figure 1).

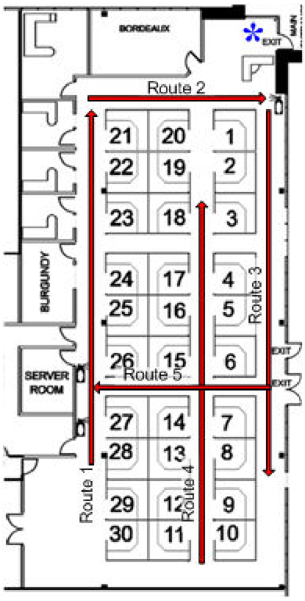

Fig. 2. Indoor route map. The blue asterisk indicates the corresponding corner of the office environment in the outdoor map.

B. Equipment

We used the RUSK Stanford channel sounder [10], which performs MIMO measurements based on the “switched array” principle [11]. The transmitter sends a chirp-like signal to sound the channel, which is eventually recorded at the receiver unit. By post-processing we then obtained the complex channel transfer function $\mathbf{H}(t, f)$.

The measurements were taken at a center frequency of 2.45 GHz with a bandwidth of 240 MHz, and a test signal length of 3.2 μs. Since we experienced occasional interference from WiFi equipments and microwave ovens, we decided to concentrate on the lowest 70 MHz of the measured spectrum in this evaluation, i.e. the band from 2.33 to 2.40 GHz.

The transmitter output power was 0.5 W. A rubidium reference in the two units ensured accurate timing and clock synchronization.

In this campaign we strived to measure routes with very high position accuracy. For this reason, we used a distance measuring wheel, providing a trigger signal every 1.6 cm ($\pm 2\%$) to the sounder. So, the channel transfer function can also be expressed as a function of distance, $\mathbf{H}(d, f)$, where d is a multiple of 1.6 cm.

At the outside location, an array of two dual-polarized WiMAX base station antennas were mounted on a scissor lift raised to a height of 10 m (see Figure 3a). The 4 antenna ports were connected by long low-loss RF cables to the transmitter on the ground. Indoors, we mounted four different types of antenna arrays onto the receiver unit (see Figure 3b): (i) two omni-directional Discone antennas with a spacing of 4.6 wavelengths (Figure 4a), (ii) two patch antennas in a WiMAX customer premises equipment (CPE, see Figure 4b), (iii) an array of two planar inverted F antennas on a PC card (Figure 4c), and (iv) an array of two ceramic antennas on a USB dongle (Figure 4d).

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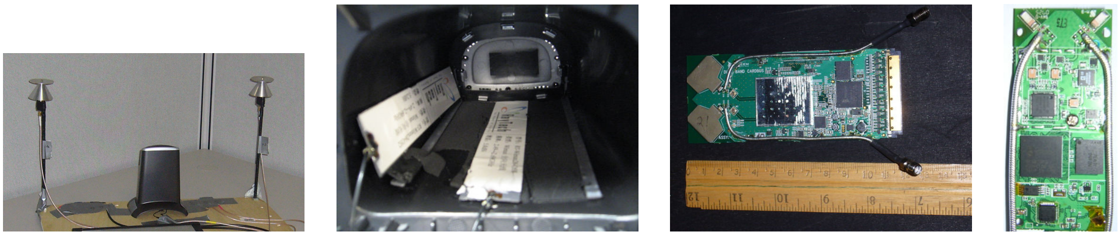
[2] The measurement data is publicly available upon request.

(a) (b)

Fig. 3. (a) Base station antennas at the transmitter lifted to 10m, (b) Receive arrays used for the O2I moving measurements



(a) (b) (c) (d)

Fig. 4. Antennas used for the outdoor-to-indoor mobile measurements: (a) Discones and CPE mounted on a wooden board, (b) inside-view of the CPE array: two patch antennas with orthogonally oriented main lobes, (c) broadband WiMAX PC-card antenna array, (d) narrowband WiMAX antenna array with ceramic elements

III. CHANNEL SIMILARITY MEASURES

In our numerical evaluations, we concentrate on the spatial correlation matrices (at the BS, and the full correlation matrix). Since we are interested in the design and analysis of multi-user MIMO prefiltering and other space-time algorithms, the similarity metrics should compare the *singular value structure* of these matrices. Taking this into account, we quantify the similarity in two ways: by the collinearity between matrices, and by their condition number.

A. Matrix collinearity

The distance between two matrices of same dimensions can be quantified by the collinearity given by [12]

$$c(\mathbf{A},\mathbf{B}) = \frac{|\mathrm{tr}(\mathbf{A}\mathbf{B}^{\mathrm{H}})|}{\|\mathbf{A}\|_{\mathrm{F}}\|\mathbf{B}\|_{\mathrm{F}}}, \qquad (1)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the (complex-valued) matrices to be compared, $\|\cdot\|_{\mathrm{F}}$ denotes the Frobenius norm of a matrix, and $(\cdot)^{\mathrm{H}}$ is the matrix conjugate transpose operation. A helpful interpretation of this measure comes from the fact that it is a normalized inner product, and thus has a geometric meaning. For example, if $\mathbf{A}$ and $\mathbf{B}$ are real-valued, then $c(\mathbf{A},\mathbf{B}) = |\cos\angle(\mathrm{vec}(\mathbf{A}),\mathrm{vec}(\mathbf{B}))|$, where $\mathrm{vec}(\cdot)$ stacks the columns of its matrix argument on top of each other.

In general, the collinearity describes how similar the subspaces of the compared matrices are. This measure ranges between zero (no collinearity, i.e. matrices are orthogonal to each other) and one (full collinearity, matrices are similar).

The beauty of this measure is that it compares both the singular values and the alignment of their associated singular vectors. A full collinearity is encountered when (i) *both* singular values and the singular vectors of the two matrices are equal (i.e. the matrices are exactly equal), or (ii) when both matrices individually have singular values that are all equal (in this case the singular vectors are inconsequential). This measure is invariant against fading or path loss as long as the multipath structure remains the same, i.e. $c(\mathbf{A},\beta\mathbf{B}) = c(\mathbf{A},\mathbf{B})$, and $c(\mathbf{A},\beta\mathbf{A}) = 1$ for any $\beta \neq 0$.

Note that the Correlation Matrix Distance [13] is a special case of this measure, but is only applicable to Hermitian matrices (in the authors' case: correlation matrices). Nevertheless, the basic idea between these two measures is the same. Hence, the conclusion in [13], stating that this measure is directly correlated to the performance of MIMO precoding schemes, remains valid.

B. Condition number

As second measure for similarity we use the ratio of the matrix condition numbers, defined as

$$\chi(\mathbf{A},\mathbf{B}) = 10\cdot\log_{10}\left(\frac{\lambda_{\max}(\mathbf{A})}{\lambda_{\min}(\mathbf{A})}\Big/\frac{\lambda_{\max}(\mathbf{B})}{\lambda_{\min}(\mathbf{B})}\right), \qquad (2)$$

where $\lambda_{\max}(\mathbf{A})$ denotes the largest singular value of the matrix $\mathbf{A}$.

In this measure, similarity between the condition numbers is indicated by values close to 0 dB, while while a mismatch of the condition number ratio is reflected by positive or negative values of χ.

IV. RESULTS

We quantify the distance between two links based on their spatial correlation matrices, where a link is defined as the MIMO channel from the BS to a single MS. Using the spatial correlation matrices for comparison, we test whether the local propagation characteristics match.

Regarding the antennas, we used all four ports at the base station antennas (two antennas times two polarizations), and the Discone antennas at the receiver because of their omnidirectionality. However, our results look similar for all other combinations.

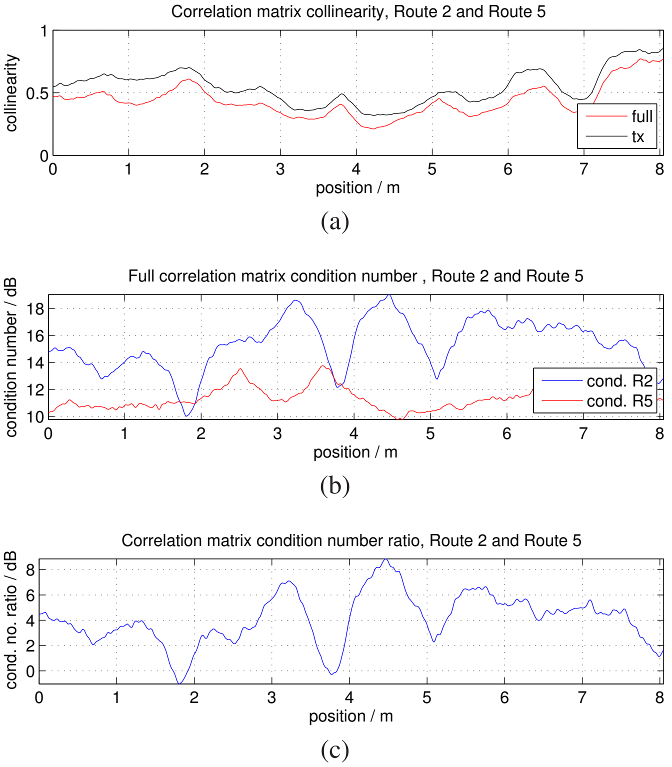
We distinguish between two kinds of spatial correlation: (i) the spatial correlation at the transmitter, $\mathbf{R}_{\mathrm{Tx}} = \mathrm{E}\{(\mathbf{H}^{\mathrm{T}})(\mathbf{H}^{\mathrm{T}})^{\mathrm{H}}\} \in \mathbb{C}^{4\times 4}$, and (ii) the full correlation matrix, $\mathbf{R}_{\mathrm{full}} = \mathrm{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\} \in \mathbb{C}^{8\times 8}$, where the $\mathrm{vec}(\cdot)$ operation stacks the columns of a matrix into a vector. We obtained the correlation matrices from the measurements by replacing the expected values above by averages in time, and frequency, namely over a sliding window of 38 snapshots (corresponding to a traveled distance of 5 wavelengths) and over all frequency samples in the interference-free band. Note that each link has its own spatial correlation properties, and thus its own correlation matrices.

We calculated the collinearity measure for both the full correlation matrices and the transmit correlation matrices, while the condition number metric is evaluated for the full correlation matrix only.

Figure 5 shows the results from comparing Route 2 with Route 5 (Tx1D1-R2 and Tx1D1-R5). These routes are well separated in the room and do not intersect. The collinearity measure (Figure 5a) shows a strong dissimilarity over all measured positions. We observe here that the *transmit correlation differs significantly between the two receiver positions*. By this finding, we *invalidate the conventional Kronecker model* for modeling multi-user environments, as proposed in [5]. In their paper, the authors assume that the transmit correlation matrix is always the same, no matter where the users are. Our data shows that this assumption does not even hold when the MSs are in the same room, and thus close to each other. We also learn from these results that the collinearity does not only depend on the condition number ratio (cf. position 3.8 m), but that it quantifies the mismatch of the subspaces spanned by the singular vectors associated with the largest singular values.

Figure 6 compares the differences of the spatial correlation along a single route. For this, we used Route 3 (Tx1D1-R3) and compared it with itself, but reversed in time (i.e. position). Of course, the resulting curves are symmetric and show that the similarity increases the closer the MSs get. Interestingly, the collinearity between the correlation matrices is higher than for the previous measurements, which we can attribute to similar propagation conditions for all positions, since the MSs were quite close to outside walls. Also note a significant difference in the condition number ratios. The condition number drops for the respective MS that is farther away from the transmitter, and thus has richer scattering around it.

Finally, we aim to compare two routes that are not in the same building. To emulate this situation, we considered measurements of a single route, collected from two different transmitter rotations (Tx1D2-R3 and Tx1D3-R3). The first

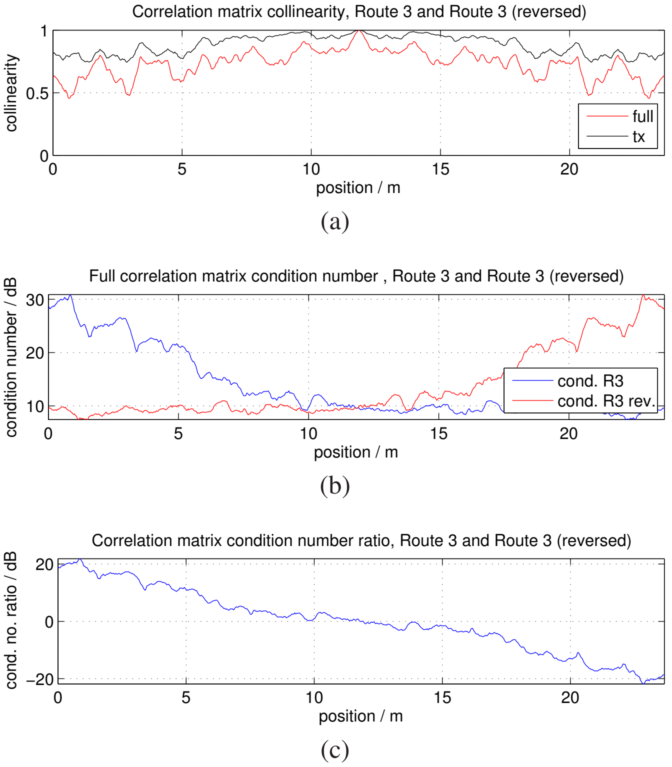


(a)

(b)

(c)

Fig. 5. Comparing the similarity of the spatial correlation matrices for Route 2 and Route 5. (a) Collinearity measure, (b) condition numbers, (c) condition number ratio

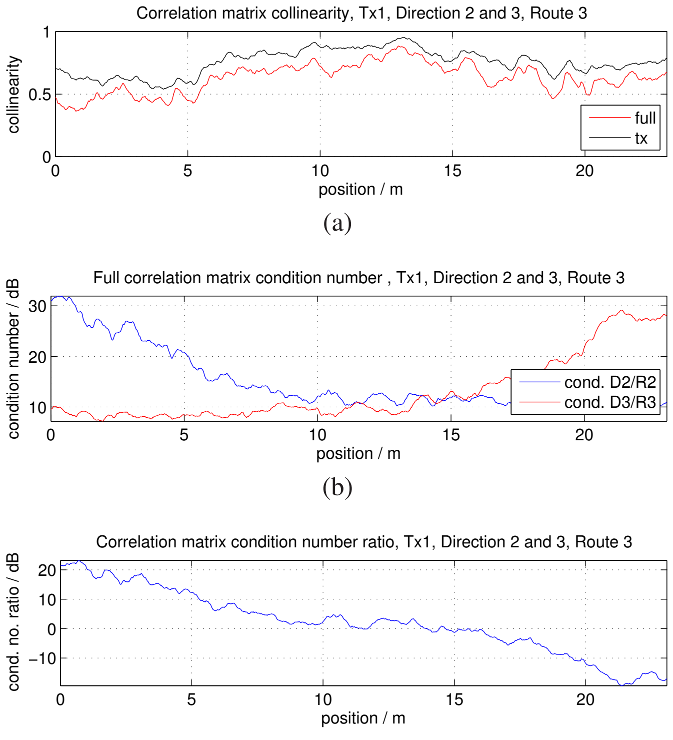


(a)

(b)

(c)

Fig. 6. Comparing the similarity of the spatial correlation matrices for Route 3 and an inverse run of Route 3. (a) Collinearity measure, (b) condition numbers, (c) condition number ratio



(a)

(b)

(c)

Fig. 7. Comparing the similarity of the spatial correlation matrices for Route 3 measured from Tx1D2 and Tx1D3. (a) Collinearity measure, (b) condition numbers, (c) condition number ratio

run of the route appears to be on the left of the transmitter, while the second run of the (same) route appears to be on the right hand side. Since we kept the environment as invariant as possible, this approach is feasible [8]. Figure 7 shows the result of this evaluation. It turns out that there is a strong dissimilarity of the correlation matrices in the beginning (the collinearity is only around 0.5 for some points of the route), but farther on the route, we observe a stronger collinearity. In this case, the collinearity is strongly linked to the condition number ratio, indicating that in this scenario the singular values have a stronger impact on the similarity than the the singular vectors.

V. CONCLUSIONS

We compared the spatial correlation matrices from multi-user MIMO outdoor-to-indoor channel measurements.

To quantify the similarity between the spatial structure of the channels, we used two similarity metrics that are susceptible to the eigenstructure of the channel: matrix collinearity and the condition number. Our measurement-based numerical results showed that both measures provide significant information about the dissimilarity of the spatial structure in a different notion. While the condition number ratio tells whether some channels are more directive, the collinearity measure is sensitive to the alignment or non-alignment of the preferred directions.

We substantiated that the spatial structure of multi-user channels can differ significantly, even when the users are in close proximity (e.g. in the same room). This finding clearly invalidates the conventional Kronecker modeling assumption (stating that the correlation structure around the base station is the same for all MS positions). Thus, new *multi-user* MIMO channel models, which include this property, are needed.

Our finding of spatially separable channels impacts future space-time algorithms. Different multi-antenna terminals (which might even be in the same room) can be separated in the spatial domain, when the channels permit. In a multi-user environment, these spatially separable channels and users can be scheduled jointly, which minimizes interference and increases the overall system capacity.

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